## Book Review

## Turbulence, Coherent Structures, Dynamical Systems and Symmetry

P. Holmes, J. L. Lumley, and G. Berkooz, Cambridge University Press, New York, 1996, 420 pp., \$69.95

This is a much-needed book in the field of turbulence. Until now there has not been a detailed one-stop source for teaching or learning modern developments in turbulence: specifically, coherent structures, strange attractors, proper orthogonal decomposition, and lowdimensional modeling. The intent of the authors is not to provide a comprehensive treatment of turbulence or dynamical systems but to address how dynamical system concepts have contributed to improved understanding of turbulent flows. The book has certainly addressed the issue. It has grown out of more than a decade of research by the authors on low-dimensional models of turbulent boundary-layer flow, and therefore, understandably, this topic is its primary focus. A brief review of other applications is also presented. Furthermore, many simple examples are provided to illustrate relevant concepts from dynamical systems. Thus this book can be used to supplement classical turbulence textbooks by Batchelor, Tennekes, and Lumley or Monin and Yaglom to introduce coherent structure and dynamical system concepts in a graduate-level course in turbulence.

The book is divided into four parts: 1) turbulence, 2) dynamical systems, 3) the boundary layer, and 4) other applications and related work. Part one begins with an introduction to turbulence and low-dimensional modeling. A glossary of notation and mathematical jargon employed is also provided. The rest of part one (Chapters 2–4) provides a brief background on turbulence, describes coherent structures (principally based on experimental observations), introduces the proper orthogonal decomposition methodology, and presents Galerkin projection of the Navier–Stokes equations onto a finite

dimensional subspace to result in a finite set of ordinary differential equations. The chapter on proper orthogonal decomposition also discusses symmetry and invariance, stochastic estimation, and some past applications. Part two contains four chapters and begins with a brief review of some of the important dynamical system concepts required for a low-dimensional projection of the Navier-Stokes equations. Simple examples are provided to illustrate structural stability, local and global bifurcations, and simple and strange attractors. An entire chapter (Chapter 6) is devoted to symmetries. In Chapter 7, the dynamical system concepts are put to the test in a simple sample problem represented by the Kuramoto-Sivashinsky equation. Part two ends with a brief investigation of the effect of noise on the evolution of the dynamical system. Part three begins (Chapter 9) with the construction of a class of low-dimensional models for boundary-layer flow, followed by a discussion of its behavior in phase space and physical implications (Chapter 10). Part four briefly presents proper orthogonal decomposition and (in some cases) low-dimensional model results for other applications such as the circular jet, forced-mixing layer, and grooved channel. The book concludes with a speculative note as to how all of these dynamical system concepts can help us to better understand turbulence.

In summary, this book will be of considerable value both in the classroom as a supplementary text and as a reference volume for current research in coherent structures, proper orthogonal decomposition, and lowdimensional representation.

> S. Balachandar University of Illinois